The system is given as $\dot{x} = ax + bu + c$.

Here $x, u, a, b, c \in \mathbb{R}$ and $b > 0$. The desired reference is $r(t) = 0$.

MRAC Design

Here $f(x) = -c/b$ which allows us to state that $\theta = -c/b$ and that $\phi(x) = 1$.

The model reference is chosen to be $\dot{x}_m = -x_m$, which means that $a_m = -1 < 0$ and that $b_m = 0$ for any $r(t)$ (although in this case the reference is null so it does not matter).

The adaptive control law is obtained as $u = \hat{k}_x x + \hat{k}_r x + \hat{\theta}^\top \phi(x)$.

Here, the parameter estimates are updated as follows,

$$\begin{cases} \dot{\hat{k}}_x = -\gamma_x x e \ \text{sgn}(b) \\ \dot{\hat{k}}_r = -\gamma_r x r \ \text{sgn}(b) \\ \dot{\hat{\theta}} = -\gamma_x \phi(x) e \ \text{sgn}(b) \end{cases}$$

Recall that $\gamma_x, \gamma_r > 0$, therefore it is permissible to set them as $\gamma_x = \gamma_r = 1$.
Recall that $b > 0$, therefore $\text{sgn}(b) = 1$, and that, $\phi(x) = 1$ as established above.
Recall that $r = 0$ and that $e = x - x_m$.
Simplify and reduce the update expressions, obtaining,

$$\boxed{\begin{cases} \dot{\hat{k}}_x = -x(x - x_m) \\ \dot{\hat{k}}_r = 0 \\ \dot{\hat{\theta}} = -(x - x_m) \end{cases}}$$

Effectively, the input is reduced to $\boxed{u = \hat{k}_x x + \hat{\theta}}$.

The closed-loop system becomes,

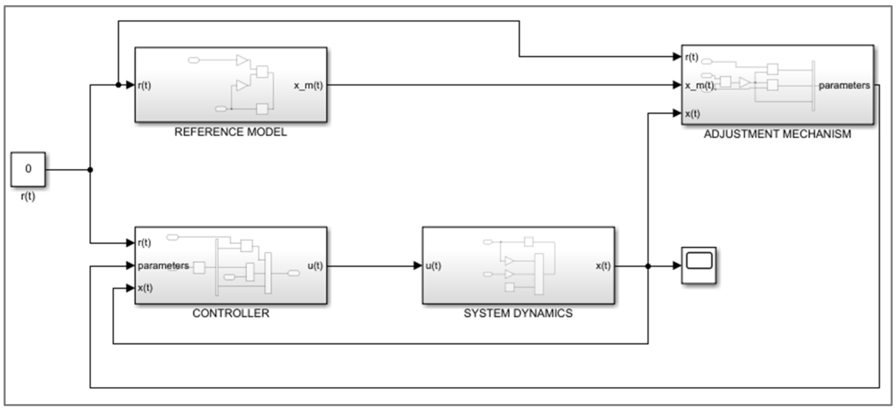
$$\begin{aligned} \dot{x} &= (a + b\hat{k}_x)x + b\left[\hat{k}_r r + \left(\hat{\theta} - \theta\right)^\top \phi(x)\right] \\ &= (a + b\hat{k}_x)x + b\left(\hat{\theta} - \theta\right) \end{aligned}$$

Simulink Implementation

Refer to `mrac.slx` *for the implementation in* `Simulink`.

The setup described is simulated in Simulink with $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3.5 \\ 1.2 \\ 3.5 \end{bmatrix}$ and $x(0) = -5$.

The overall control loop is,



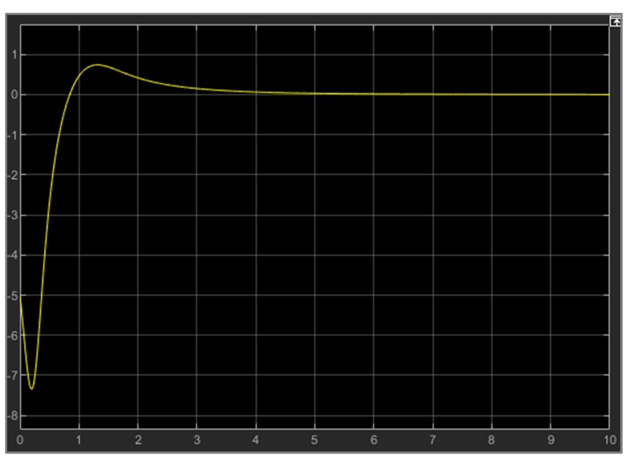
Here, `CONTROLLER` contains outputs the adaptive control law $u(t)$.

`SYSTEM DYNAMICS` contains the first-order equation of the system with the parameters and initial condition properly set.

`REFERENCE MODEL` contains the reference dynamical system model set to $\dot{x}_m = -x_m$.

`ADJUSTMENT MECHANISM` updates the values for the parameters $\hat{k}_x$, $\hat{k}_r$, and $\hat{\theta}$ according to the update expressions derived in page 2.

The reference signal is null, and the scope captures the state $x(t)$, plotted below.



The controller functions as intended, and the state converges to zero.